

Mental Math Solutions

CHILES MINI MU

2023-2024

1. Compute the sum of the first 20 natural numbers.

Solution. We compute $1 + 2 + \cdots + 20 = \frac{1}{2} \cdot 20 \cdot 21 = \boxed{210}$. \square

2. Given positive integers a, b, c that satisfy $29a + 30b + 31c = 366$, compute $a + b + c$.

Solution. We have

$$\frac{366}{31} = \frac{29}{31}a + \frac{30}{31}b + c < a + b + c < a + \frac{30}{29}b + \frac{31}{29}c = \frac{366}{29},$$

so $a + b + c = \boxed{12}$. This can be approximated easily as $\frac{366}{29} \approx \frac{366}{31} \approx \frac{360}{30} = 12$. Alternatively, one can note that a, b, c represent months in a leap year, and there are twelve months in a year. \square

3. What is 12% of 3/4ths of 600?

Solution. 3/4ths of 12% is 9%, so the answer is $6 \cdot 9 = \boxed{54}$. \square

4. Given that 6 second-round picks are equal to 1 role player, 3 first-round picks are equal to 1 star player, and 2 first-round picks are equal to 3 role players, how many second-round picks are equal to one star player?

Solution. One star player equals 3 first-round picks, which equals $3 \cdot \frac{3}{2} = \frac{9}{2}$ role players, which equals $\frac{9}{2} \cdot 6 = \boxed{27}$ second-round picks. \square

5. Aaron has a zoo that only has monkeys, which have 2 legs, and bears, which have 4 legs. If his zoo has 40 animals that have a total of 100 legs, how many monkeys are in his zoo?

Solution. If all the animals were bears, there would be $40 \cdot 4 = 160$ legs, Replacing a bear with a monkey reduces the total amount of legs by 2, so the total amount of monkeys is $(160 - 100)/2 = 60/2 = \boxed{30}$. \square

6. Compute the minimum possible value of $2x^2 + 4x + 7$ over reals x .

Solution. We complete the square: $2x^2 + 4x + 7 = 2(x + 1)^2 + 5$, so the answer is $\boxed{5}$. \square

7. What is the slope of the line that passes through (24, 7) and the vertex of the parabola in the above question?

Solution. The minimum is attained at $x = -1$, so the vertex is $(-1, 5)$. Then the slope is $\frac{7-5}{24-(-1)} = \boxed{\frac{2}{25}}$. \square

8. Compute the sum of the first 24 whole numbers.

Solution. We compute $0 + 1 + \cdots + 23 = \frac{1}{2} \cdot 24 \cdot 23 = 12 \cdot 23 = \boxed{276}$. □

9. What is the ratio of the volume of a cylinder with radius 50 and height 125 to the volume of a cylinder with radius 100 and height 25 expressed as a common fraction?

Solution. The ratio of the volumes is $\left(\frac{50}{100}\right)^2 \cdot \frac{125}{25} = \left(\frac{1}{2}\right)^2 \cdot 5 = \boxed{\frac{5}{4}}$. □

10. Linsey goes to sleep at 3 : 17 PM and sets her alarm to wake her up at 6 : 00 PM. How many minutes does she sleep for?

Solution. She sleeps 17 minutes less than 3 hours = 180 minutes, so the answer is $180 - 17 = \boxed{163}$. □

11. What is the number of the distinct prime factors of $5^3(5^3 - 1)$?

Solution. The only prime factor of 5^3 is 5. Additionally, $5^3 - 1 = 124 = 4 \cdot 31$. Thus, the product has distinct prime factors 2, 5, and 31, so the answer is $\boxed{3}$. □

12. How many ways are there to label the vertices of a hexagon with A, B, C, D, F, F if configurations that are rotations of each other are considered to be the same?

Solution. If the labels were all distinct, the answer would be $(6 - 1)! = 120$. However, since there are two F 's we must divide by 2 for double counting, so the answer is $120/2 = \boxed{60}$. □

13. Compute 88×99 .

Solution. $88 \cdot 99 = 88(100 - 1) = 8800 - 88 = \boxed{8712}$. □

14. What is the area of the triangle with side lengths 20, 21, and 29?

Solution. Note that $20^2 + 21^2 = 400 + 441 = 841 = 29^2$, so the triangle is right, and the answer is $\frac{1}{2} \cdot 20 \cdot 21 = \boxed{210}$. □

15. Shaoyang gives his dog 3 treats if he rolls over and 5 treats if he does a flip. If his dog rolls over 17 times and does 38 flips, how many treats will Shaoyang give him?

Solution. The answer is $3 \cdot 17 + 5 \cdot 38 = 51 + 190 = \boxed{241}$. □

16. What is the total number of sides in a pentagon, hexagon, and octagon?

Solution. This is $5 + 6 + 8 = \boxed{19}$. □

17. Compute the sum of all distinct possible areas of a square that has two vertices at (20, 24) and (24, 20).

Solution. The distance between these two points is $4\sqrt{2}$, which is either the length of the side length of the length of the diagonal of the square. If it is the side length, the area is $(4\sqrt{2})^2 = 32$, and if it is the diagonal, the side length is 4 and the area is 16. Then the answer is $32 + 16 = \boxed{48}$. □

18. Find the positive integer solution to $x^2 = 7225$.

Solution. Note that $80^2 = 6400 < 7225 < 8100 = 90^2$, so $80 < x < 90$. Additionally, $5 \mid 7225 = x^2$, so $5 \mid x$, and the answer must be $\boxed{85}$. You can also think of the trick when squaring a number that ends in 5. □

19. Compute the product of all integers from -20 to 24 inclusive.

Solution. This product contains 0, so the answer is just $\boxed{0}$. \square

20. Yimo has 2 cards labeled 2, 3 cards labeled 3, 4 cards labeled 4, and so on until 10 cards labeled 10. He shuffles the cards and begins randomly drawing them one by one. What is the smallest amount of cards Yimo needs to draw to guarantee that he will have at least one card of every label?

Solution. The hardest card to guarantee that he has are the cards labeled 2, as those have the least amount. Then he needs to draw at least

$$(3 + 4 + \cdots + 10) + 1 = (1 + 2 + \cdots + 10) - (1 + 2) + 1 = \frac{10 \cdot 11}{2} - 2 = 55 - 2 = \boxed{53}$$

cards. \square

21. What is the remainder when $17 \cdot 23$ is divided by 40?

Solution. We have $17 \cdot 23 = 20^2 - 3^2$, and 20^2 is clearly divisible by 40. Then the answer is $40 - 3^2 = \boxed{31}$. \square

22. Compute $1 - 2 + 3 - 4 + \cdots - 2022 + 2023$.

Solution. For all terms but the last, we can pair consecutive terms to equal -1 . Then the answer is $2023 - 2022/2 = 2023 - 1011 = \boxed{1012}$. \square

23. In chess, pawns are worth 1 point, knights and bishops are worth 3 points, rooks are worth 5 points, and queens are worth 9 points. How many total points are 4 pawns, 3 knights, 3 bishops, 3 rooks, and 2 queen worth?

Solution. The answer is $3(1 + 3 + 3 + 5 + 9) + 1 - 9 = 3 \cdot 21 - 8 = 63 - 8 = \boxed{55}$. \square

24. What is the number of distinguishable permutations of *IRVING*?

Solution. This has 6 letters with one duplicated letter, so the answer is $6!/2! = \boxed{360}$. \square

25. Hadriel randomly chooses a factor of 60. What is the probability that he chooses a composite number?

Solution. We can factor $60 = 2^2 \cdot 3 \cdot 5$, which has $3 \cdot 2 \cdot 2 = 12$ factors. 60 has three prime factors, so it has $12 - 3 - 1 = 8$ composite factors, where we subtracted 1 to account for 1. Then the answer is $\frac{8}{12} = \boxed{\frac{2}{3}}$. \square

26. What is the fifth prime number?

Solution. Listing primes, we see that the fifth prime number is 2, 3, 5, 7, $\boxed{11}$. \square

27. Today is December 9, 2023, which is a Saturday. Find x , where December x , 2024 is the first Saturday in December of 2024.

Solution. Noting that 2024 is a leap year, in $366 \equiv 2 \pmod{7}$ days after December 9, 2023, it will be December 9, 2024, a Monday. Then December 7, 2024, is a Saturday, and it is clearly the first, so the answer is $\boxed{7}$. \square

28. What is the smallest positive integer such that when multiplied by 2023 it is a perfect square?

Solution. Note that $2023 = 7 \cdot 17^2$, so the answer is clearly $\boxed{7}$. \square

29. Compute $77^2 - 23^2$.

Solution. $77^2 - 23^2 = (77 + 23)(77 - 23) = 100 \cdot 54 = \boxed{5400}$. □

30. Nonoko has \$2024 in the form of \$1 and \$2 bills. If she has at least one kind of each bill, then what are the number of possible distinguishable combinations of bills Nonoko could have?

Solution. She can have any amount from 1 to $2024/2 - 1 = 1011$ of \$2 bills, with \$1 bills covering the rest of the total amount. Then the answer is $\boxed{1011}$. □

31. How many ways are there to choose a committee of 2 men and 5 women from a group of 6 men and 6 women?

Solution. The answer is $\binom{6}{2}\binom{6}{5} = 15 \cdot 6 = \boxed{90}$. □

32. Aaron, Bob, Charley, Darius, and Eddie sit in a row of 5 seats in a movie theater. How many ways can they sit if Darius is allergic to Aaron and refuses to sit next to him?

Solution. There are $5!$ ways for them to sit without restriction. To count the ways we don't want, where Darius and Aaron sit together, there are $4!$ ways to arrange the four objects AD , B , C , and E , and 2 ways to permute A and D (either AD or DA). Thus, the number of ways we want is $5! - 2 \cdot 4! = 4!(5 - 2) = 24 \cdot 3 = \boxed{72}$. □

33. Let $x \star y = x^y + y^x + xy$. Compute $2 \star 3$.

Solution. We can compute $2 \star 3 = 2^3 + 3^2 + 2 \cdot 3 = 8 + 9 + 6 = \boxed{23}$. □

34. Using the notation from the previous question, compute $0 \star (1 \star (2 \star \dots (9 \star 10) \dots))$.

Solution. Note that $0 \star x = 0^x + x^0 + 0 \cdot x = 0 + 1 + 0 = 1$, as long as x is nonzero. Then the answer is clearly $\boxed{1}$. □

35. David has 5 fair coins each with a 1 on one side and a 0 on the other side. He flips these coins and then rolls a fair six-sided die with faces labeled 1 through 6. What is the probability that the sum of the numbers that turn up on the coins and the die is 6?

Solution. Regardless of the results of the coins, there is a $\frac{1}{6}$ chance that the right number will be rolled on the die, so the answer is $\boxed{\frac{1}{6}}$. □

36. How many three-digit positive integers exist where all their digits are divisible by 3?

Solution. The leading digit can be 3, 6, or 9, and the second and third digits can be 0, 3, 6, or 9, and the answer is $3 \cdot 4 \cdot 4 = \boxed{48}$. □

37. What is the length of the longest chord of integer length that can be drawn in the circle that circumscribes a square with side length 8?

Solution. The diameter of the circle is equal to the diagonal of the square, which is $8\sqrt{2} \cdot 1.4 = 11.2$. Then the answer is $\lfloor 8\sqrt{2} \rfloor = \lfloor 11.2 \rfloor = \boxed{11}$. □

38. Compute the sum of all integers from -20 to 24 inclusive.

Solution. Everything from -20 to 20 cancels, and we are left with $21 + 22 + 23 + 24 = \boxed{90}$. □

39. What is the slope of the line $20x + 24y = 2024$?

Solution. Rearranging and dividing both sides by 24 gives $y = -\frac{20}{24}x + \frac{2024}{24}$, so the answer is $-\frac{20}{24} = \boxed{-\frac{5}{6}}$. \square

40. While practicing chemistry, Nima learns that Avogadro's number is approximately $n = 6 \times 10^{23}$. How many positive integer factors does n have?

Solution. We can factor $n = (2 \cdot 3)(2 \cdot 5)^{23} = 2^{24} \cdot 3^1 \cdot 5^{23}$, so the answer is $25 \cdot 2 \cdot 24 = \boxed{1200}$. \square